# Topics in Optimization

## (Entropy-Maximization and Variational Methods)

(22-672 – Spring 2021)

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**Level:** Last year Undergraduate/Graduate

**Prerequisites:** Linear Algebra, Probability Theory, Basic Programming.

**Course description:** The main objective of this course is to introduce the general paradigm of algorithm design which is related to entropy-based variational method. This approach which has been introduced and considerably developed within the past 20 years, has its roots in statistical mechanics [6, 7], while its recent applications in statistical inference within the context of graphical models has turned it into a very powerful tool to be used in a variety of statistical, computational and mathematical fields with applications in machine learning and artificial intelligence, bioinformatics, communication theory, combinatorial optimization, signal and image processing, mathematical finance and analysis of many other complex systems (e.g. see [1, 6, 9, 10]).

The strong point of this dual-optimization approach lies in applicability of a culmination of ideas borrowed from physics, Bayesian inference and convex optimization, where as an inference method, this approach can essentially be characterized as an alternative to the MCMC or other algorithmic paradigms which are based on sampling techniques.

The main objective of the course is to present the basic theoretical facts, ideas and techniques needed to design such algorithms with an emphasis on implementations. A schedule of the course topics are as follows:

- **A probabilistic toolbox, concentration of measure and isoperimetry [2, 4, 6].**
- **Basics of information theory and statistical physics [2, 5, 6].**
- **Foundations of the variational method [2, 3, 9].**
- **Introduction to cavity method and belief propagation [5, 6, 10].**
- **Mean-field approximations [8, 9].**
- **Linear approximation of the cavity method, nonbacktracking matrix and the stochastic block model [6, 10].**
- **Foundations of graphical models [8, 9].**
- **Projects.**

**Evaluation:** will be based on the student's performance in doing the exercises, final and take-home exams along with an implementation project.

## References

[1] Jean-Philippe Bouchaud, Marc Mézard, and Jean Dalibard, editors. *Complex Systems*, volume 85 of *Les Houches*. Elsevier B.V., 2007.

[2] S. Boucheron, G. Lugosi, and P. Massart. *Concentration Inequalities: A Nonasymptotic Theory of Independence*. OUP Oxford, 2013.

[3] S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.

[4] R. van Handel. *Probability in High Dimension*. APC 550 Lecture Notes. Now Publishers, 2016.

[5] O. C. Martina, R. Monasson, and R. Zecchina. Statistical mechanics methods and phase transitions in optimization problems. *Theoretical Computer Science*, 265:3–67, 2001.

[6] M. Mézard and A. Montanari. *Information, Physics, and Computation*. Oxford Graduate Texts. OUP Oxford, 2009.

[7] H. Nishimori. *Statistical Physics of Spin Glasses and Information Processing: An Introduction*. Oxford University Press, 2001.

[8] S. Ravanbakhsh. *Message Passing and Combinatorial Optimization*. PhD thesis, Department of Computing Science, 2015.

[9] M. J. Wainwright and M. I. Jordan. *Graphical Models, Exponential Families, and Variational Inference*. Foundations and trends in machine learning. Now Publishers, 2008.

[10] L. Zdeborová and F. Krzakala. Statistical physics of inference: thresholds and algorithms. *Advances in Physics*, 65(5):453–552, 2016.